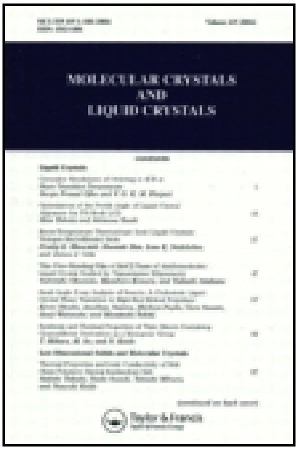

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On Model Independent Restoration of the Surface Anchoring Potential

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On Model Independent Restoration of the Surface Anchoring Potential

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An analysis of various options to restore model independently the actual shape of the surface anchoring potential from the experimental measurements at chiral liquid crystals (CLC) is performed. It is shown that if the surface anchoring is sufficiently weak the restoration of the actual shape of the surface anchoring potential is achievable and there are some options to do this. The simplest option relates to the optical measurements at homogeneous layers of CLC under variation of some external parameter. Another possibility is connected with studying of the temporal development of the pitch jumps which occur in a homogeneous layer at some critical value of the external parameter. Additional options are presented by layers with nonsingular wall in the director distribution dividing two area differing by the pitch value: the director distribution in the wall is directly dependent on the surface anchoring potential. If a wall dividing two areas differing by the pitch value moves the motion velocity is also dependent on the surface anchoring potential and the corresponding measurements also may be applied for the restoration of the surface anchoring potential. The formulas for deriving of the angular derivative of the surface anchoring potential from the measurements related to the mentioned options are presented. The angular range of this derivative determination for the various studied options is estimated. The available experimental data are used to estimate the potential derivative and compare it with the derivative for the known model surface anchoring potentials.

Keywords: director dynamics; nonsingular walls; pitch jumps; surface anchoring

INTRODUCTION

Recently the problem of restoration of the actual shape of the surface anchoring potential [1] for liquid crystals (LC) again has attracted interest of the researchers [2–4]. It was partially due to the achievements

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in obtaining of a very low surface anchoring [5,6] and to the development of a new direct method of visualization of the director distribution in bulk of liquid crystals [7,8]. There appeared also indications that the shape of widely used the Rapini–Papoular (R–P) model surface anchoring potential [9] does not agree with the results of rigorous theoretical study of the problem [10]. The surface anchoring reveals itself in jumps and in hysteresis of the director distribution in LC under continuous variations of some external agents applied to LC. These may be the temperature [11], applied electric or magnetic field [12], mechanical twisting of LC [13], thickness of the LC layer [14] and so on.

Traditionally [15] it is accepted that the surface anchoring is described by the Rapini–Papoular (R–P) model surface anchoring potential [9]. If the anchoring is strong and, consequently, the director deviations from the easy direction are small the R–P potential suites excellently for the description of the surface anchoring because it is quadratic on the director deviation angles at small deviation angles (as it should be for any reasonable surface anchoring potential). However, if the deviation angles are large it may be not the case. Just the mentioned above director jumps may occur at large director deviations from the easy direction, where the surface anchoring potential may be not quadratic on the director deviation angles and, in particular, may differ from the R–P potential.

It happens that if the anchoring is sufficiently weak (more accurate, some dimensionless parameter of the problem is sufficiently large) the mentioned phenomena become sensitive to the shape of the surface anchoring potential. A similar sensitivity to the shape of actual surface potential is revealed by the director distribution in nonsingular walls in LC layers [4] and their motion in LC layers [4,16]. The mentioned above jumps of the director distribution in LC, naturally, are processes with temporal development occurring during finite time which duration and temporal development, as was shown in [3], are also dependent on the shape of the actual surface potential. The most natural mechanism of the jump-like director configuration transformations at weak anchoring, which is assumed here, is director slipping at the surface over the potential barrier of anchoring forces [17,18]. At strong anchoring the jump-wise transformation may proceed via formation of defects in the director distribution in LC and, in particular, via director deviations from the plane of the original director distribution [1]. Below possible ways to restore the actual surface anchoring potential from experiments at weak surface anchoring are analyzed and some recommendations for the corresponding possible experiments are presented.

STATIC DIRECTOR CONFIGURATIONS

Plane layer of fixed thickness. At the beginning consider static director configurations in LC and their relation to the surface anchoring potential. For definiteness consider a plane cholesteric layer with an infinite anchoring at one its surface and finite anchoring at the second one and following to [17] take as a variable parameter the temperature T. The expression for the free energy of a homogeneous layer accepts the following form

$$F(T) = W_s(\varphi) + (K_{22}/2d)[\varphi - \varphi_0(T)]^2, \tag{1}$$

where φ is the director deviation angle from the alignment (easy) direction at the surface with finite anchoring, $W_S(\varphi)$ is the surface anchoring potential, K_{22} is the elastic twist modulus, d is the layer thickness. The angle φ_0 (T) gives the angle of the director deviation from the alignment direction at the surfaces with finite anchoring if the anchoring at this surface were absent at all, i.e., the free rotation angle determined by the temperature variations of the pitch in a bulk cholesteric. Note that the surface anchoring potential W_s is dependent generally on the azimuthal and polar angles [1]. In our case only the dependence of the anchoring potential on the azimuthal angle φ is essential so it will be assumed below that the polar angle does not change at all and the surface anchoring potential W_s depends on φ only. The above assumption is definitely acceptable if the polar anchoring is much stronger than the azimuthal one (what is quite frequently the case [5,6]).

The equation determining the director deviation from the easy direction at the surface with finite anchoring may be easily found [17]:

$$\partial w(\varphi)/\partial\varphi + S_d[\varphi - \varphi_0(T)] = 0, \tag{2}$$

where, to conserve the previous notations [2–4,13,14,19–21], we present the surface anchoring potential in the standard form (like model Rapini–Papoular [9] and B-potentials [4,20]) introducing a normalized function $w(\varphi): W_s(\varphi) = Ww(\varphi)$, where W (the doubled depth of the anchoring potential well) is some constant of surface energy dimensionality, $w(\pi/2) - w(0) = 1/2$ and the dimensionless parameter $S_d = K_{22}/Wd$. Figure 1 presents $w(\varphi)$ for Rapini-like (R–P) and B-like (B) model serfuce anchoring potentials.

If one accepts that $\varphi_0(T)$ is known (from the pitch temperature dependence for a bulk cholesteric) experimental measurements of the angle φ as a function of φ_0 allow to determine the angular derivative of the surface anchoring potential $dW_s(\varphi)/d\varphi$ using Eq. (2) (it is naturally assumed that the elastic twist modulus K_{22} and the layer

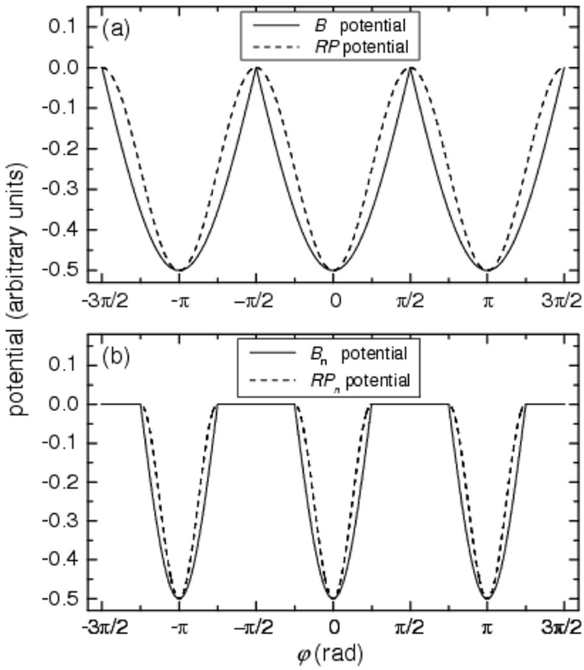

FIGURE 1 Model anchoring potentials: Rapini–Papoular and B-potentials (a) and narrow Rapini–Papoular and B-like potentials (b).

thickness d are known). Because the anchoring potential is, in principle, determined relative to the arbitrary level, i.e., it may include an arbitrary constant, the measured angular derivative of the surface anchoring potential (or the difference of the potential for two values of φ) is complete information accessible from the experiment on the surface anchoring potential. Restoration of the potential itself assumes performing of integration of the measured data (and adding of the mentioned constant relevant to the specific experimental situation). For example, in the experiments [11,13] the measurements of the angle φ only were performed and calculation of the angular derivative of the surface anchoring potential was not done, may be, due to the believe that the actual surface anchoring potential is presented by Rapini–Papoular potential [9].

Mechanical twisting of a layer. May be, from the experimental point of view, an option of surface anchoring potential restoration connected with a mechanical rotation of the plate limiting a cholesteric layer around of the layer normal [13] looks as more simple.

The corresponding free energy dependent on the plate rotation is given by the expression

$$F(\varphi_0) = W_s(\varphi) + (K_{22}/2d)[\varphi_0 + \varphi]^2, \quad (3)$$

where φ_0 is the plate rotation angle (In the derivation of Eq. (3) it was assumed that the temperature is constant in the course of the plate rotation and at the beginning of the rotation the director was oriented along the easy direction, i.e., $\varphi_0(T) = 0$ in Eq. (2). Under this assumption the angle φ may be found from the conditions of minimum of the free energy (3) what gives the following equations for φ:

$$\partial w(\varphi)/\partial\varphi + S_d(\varphi + \varphi_0) = 0 \quad (4)$$

Now the angle $\varphi + \varphi_0$ determines an additional rotation of the director at the surface caused by the plate rotation. In the case of nematic [13], for example, $\varphi + \varphi_0$ is simple the twist of nematic due to the rotation of a plate.

The analysis of the Eq. (4) shows that a smooth changing of the director deviation angle from the easy direction φ and, consequently, a smooth changing of the additional rotation of the director at the surface are possible while the modulus φ is less than some critical angle φ_c determining the instability point relative to a cholesteric spiral twisting of the initial director configuration. Upon achieving by φ of the critical value φ_c a jump-like change of the pitch occurs and the transition to a new configuration of the director in the layer differing by one in the number of the director half-turns N in the layer occurs.

The critical angle φ_c is determined by the solution of Eq. (4), which also satisfies the following relation

$$\partial^2 w(\varphi)/\partial^2\varphi + S_d = 0. \quad (5)$$

As it is known [3,17] the value of critical angle in the general case is dependent on the shape of anchoring potential and the anchoring strength at the both surfaces.

So, the restoration of the actual surface anchoring potential up to the critical angle φ_c at the case of mechanical rotation of the plate may be performed by insertion of the measured values φ and φ_0 in Eq. (4). Really, as was mentioned above, the Eq. (4) determines the derivative of the anchoring potential and if S_d is known the potential itself may be restored by integration of the found derivative. Otherwise the measured φ versus φ_0 allows restore the shape of the potential, i.e., the potential multiplied by an unknown factor. Figures 2–4 present the measured dependence of the nematic layer twist versus rotation of the plate limiting the layer [13].

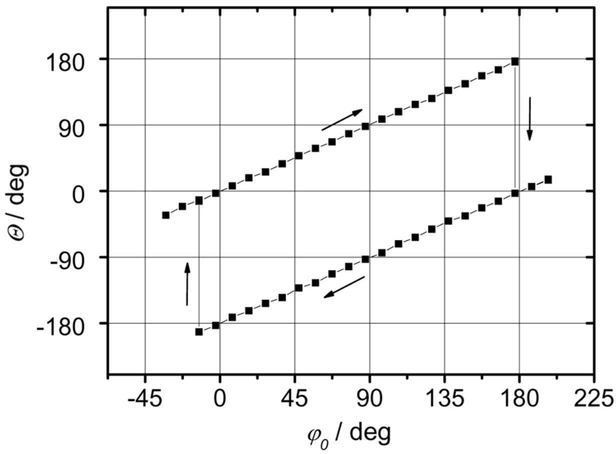

FIGURE 2 Twisting of nematic layer (θ is the director angle at surface relative to the initial easy direction) versus plate rotation in the case of strong anchoring on both walls. Sample thickness 20.5 μm [13].

Layer of variable thickness. We shall study below the restoration of the actual surface anchoring potential when a varying external agent is the thickness of the planar CLC layer [14]. The simplest experimental way to study what is happening with the director distribution

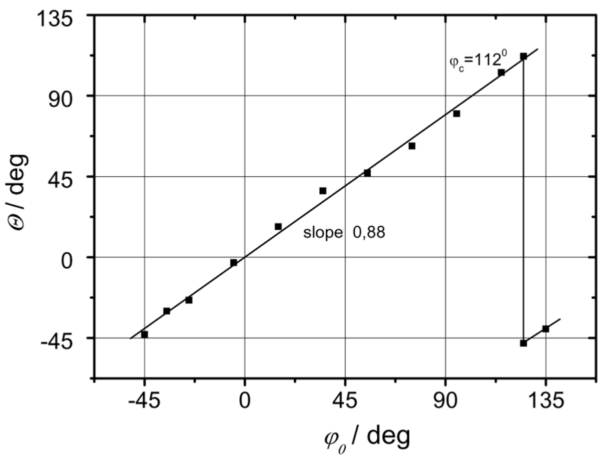

FIGURE 3 Director twist as function of the plate rotation in the case of weak anchoring on one plate and strong anchoring on the other. Sample thickness is 6 μm [13].

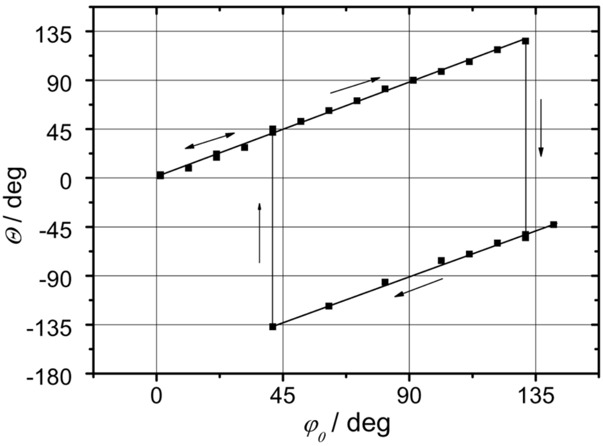

FIGURE 4 Hysteresis of the director twist θ in the case of weak anchoring at one of the plate. Sample thickness is 6 μm [13].

in the layer at continuous variation of the layer thickness is investigation of the phenomenon in a wedge cell.

Before proceed to a wedge shape sample examine the restoration procedure for a planar cholesteric layer with a finite strength of the surface anchoring under the changing of its thickness and fixed all other parameters of the problem. Specifically, we shall examine again the case of a planar layer with a finite anchoring strength at one of its surfaces and infinite anchoring at the other. Assume that the alignment directions are coinciding at the both surfaces and, as above, assume that the pitch jump mechanism is connected with the director overcoming the anchoring barrier at the surface.

The free energy of a homogeneous layer is presented again by Eq. (1) however the equation determining φ accepts the following form (different from (2)):

$$\partial \mathrm{w}(\varphi)/\partial\varphi + (\mathrm{S_d}\varphi - 2\pi\,\mathrm{l_p}) = 0. \tag{6}$$

Note that the parameter $\mathrm{S_d}$ is fixed for the problem with unchanged layer thickness and is variable in the present case, so a new dimensionless parameter $\mathrm{l_p} = \mathrm{L_p/p}$ was naturally introduced [14] in Eq. (6), where $\mathrm{L_p} = \mathrm{K_{22}/W}$ is, so called, the penetration length [1] and p is the pitch value for a bulk CLC.

Like in the case of a fixed layer thickness the solution of Eq. (6) is a smooth function of the thickness d (or parameter $\mathrm{S_d}$) in some ranges of the thickness d with abrupt jumps of φ at definite thicknesses of the layer for which φ reaches the critical value φ_c. So, the derivative of the potential $dw(\varphi)/d\varphi$ may be found from Eq. (6) up to the angle φ_c via substitution of l_p, S_d and the measured values of φ for each value of the thickness d in this equation.

The simplest way to change the thickness in the experiment is to perform the experiment at a wedge shape cell. If the wedge angle is small enough the formulas for a layer may be locally applied to the wedge. It means that Eqs. (1, 6) locally held for a wedge and the potential restoration procedure is the same as for a layer. It means that one has to measure φ as a function of the distance from the apex of the wedge and for determination of $dw(\varphi)/d\varphi$ substitute l_p, S_d and the measured value of φ in Eq. (6). Note that the range of $dw(\varphi)/d\varphi$ determination in a wedge is less than in a layer and is limited not by the angle φ_c but the equilibrium angle φ_e, the angle corresponding to the position of the wall in a wedge, i.e., limiting the Cano-Granjean zone, which is less than the angle φ_c and is decreasing with increasing of the distance from the apex of the wedge for consecutive Cano-Granjean zones [14]. The walls dividing Cano-Granjean zones in the wedge [1] demand a special examination on their informativity on the surface anchoring potential (see below).

If the parameter S_d or l_p are known the measured values of φ versus φ_0 determine $dW_s(\varphi)/d\varphi$ for the actual surface anchoring potential. However the situation which demands determination of the potential shape simultaneously with S_d or l_p in the same experiment seams as a more typical.

In the case of constant layer thickness d the quantity $(\partial w(\varphi)/\partial\varphi)/S_d$ may be determined from the measured φ and φ_0 via Eq. (2). The shape of potential, i.e., the quantity $w(\varphi)/S_d$, may be obtained by integration of the found $(\partial w(\varphi)/\partial\varphi)/S_d$. The found by integration function differs by a constant coefficient $1/S_d$ from the function $w(\varphi)$ which we have to determine. However a precise experimental determination of S_d demands the knowledge of $(\partial w(\varphi)/\partial\varphi)/S_d$ in the whole interval of φ variations $-\pi/2 < \varphi < \pi/2$. So, a practical way of the $W_s(\varphi)$ restoration if the angular shape of potential is known only in a part of the interval $-\pi/2 < \varphi < \pi/2$ assumes an approximate determination of S_d with further refining of this quantity.

As a first step at this way one may use the potential shape for a small φ which is quadratic in φ, i.e.,

$$w(\varphi) = k\ \varphi^2/2, \tag{7}$$

where the value of coefficient k is different for possible different surface anchoring potentials. Substituting (7) in the Eq. (2) on obtains for the small φ the following linear dependence of φ on the free rotation angle $\varphi_0(T)$:

$$\varphi = \varphi_0(T)/(1 + k/S_d). \tag{8}$$

So, the inclination of the measured line in the dependence of φ on the angle $\varphi_0(T)$ at small φ determines $S_d = k/(\varphi_0(T)/\varphi - 1)$. To get the approximate value of S_d one has to accept the reasonable value for k. To estimate the possible reasonable choice for k note that for Rapini–Papoular model surface anchoring potential $k = 1$ and for B-potential $k = 1/2$ [3,20].

Another option to refine S_d and to check consistency of the actual surface anchoring potential restoration for a layer of fixed thickness is exploiting of the equation for the critical angle φ_c [20].

$$\varphi_0(T_c) - \varphi_c = (\partial w(\varphi)/\partial\varphi)/S_d \tag{9}$$

at the jump point just before the jump, i.e., at $\varphi = \varphi_c$, and the equation for the angle φ_j [19], i.e., the value of φ after the jump,

$$\varphi_0(T_c) - \varphi_j - \pi = (\partial w(\varphi)/\partial\varphi)/S_d \tag{10}$$

just after the jump, i.e., at $\varphi = \varphi_j$.

The resulting equation checking consistency of the shape determination procedure does not contain S_d directly and takes the form:

$$(\partial w(\varphi)\partial\varphi)_{\varphi=\varphi_c}\big/(\partial w(\varphi)/\partial\varphi)_{\varphi=\varphi_j} = (\varphi_0(T_c) - \varphi_c)\big/(\varphi_0(T_c) - \varphi_j - \pi). \tag{11}$$

In the case of varying layer thickness d a similar estimate of l_p may be obtained via the inclination of line in a linear dependence of φ on d near to the points corresponding to $\varphi = 0$. The corresponding equation has the following form:

$$\varphi(d) = [2\pi S_{di}/(k + S_{di})](d - d_i)/p, \tag{12}$$

where S_{di} is the value of parameter S_d for the middle of N_i Cano-Granjaen zone (or the layer thickness $d_i = pN_i/2$), i.e., $S_{di} = (2/N_i)l_p$, where N_i is the number of Cano-Granjaen zone.

As an option to refine l_p and to check the estimated value of l_p consistency the relation between the coefficients in (12) for the neighboring Cano-Granjaen zones may be exploited:

$$A_{i+1} = A_i/(1 + 1/N_i)[1 - A_i/2\pi(N_i + 1)], \tag{13}$$

where $A_i = 2\pi S_{di}/(k + S_{di})$.

As was already mentioned, at measurements in a wedge the range of the potential restoration is limited by φ_e. Because φ_e is decreasing with increasing of the Cano-Granjean zone number (local thickness of the wedge) a maximal range of the actual surface anchoring potential restoration may be achieved by the described above restoration procedure in a wedge between first and second Cano-Granjean zones.

Nonsingular wall. As one saw above at smooth variations of the director orientation in one Cano-Granjean zone it is possible to restore the anchoring potential only at a limited range of director deviation from the alignment direction. However there is a principle opportunity to restore the potential in the missing range of director deviation from the alignment direction by measurements on the director distribution in a nonsingular wall between Cano-Granjean zones where the director orientation is changing fast enough however remains to be continues. Nonsingular walls may exist between several first consecutive Cano-Granjean zones if the anchoring strength is weak enough (dimensionless parameter S_d is sufficiently large) [4]. It should be noted here that the term "relatively weak surface anchoring" really is related to the large values of the dimensionless parameter $S_d = K_{22}/Wd$, where K_{22} is the elastic twist modulus, d is the layer thickness and W is the depth of the surface anchoring potential. So, at any strength of the anchoring a sufficiently thin layer (small d) insures the conditions of "relatively weak surface anchoring".

To determine the potential in the missing range $\varphi_e < \varphi < \pi/2$ one may exploit Eq. (1.13) in [4] connecting the coordinate derivative of φ in the wall with the coordinate variation of the local free energy in the wall. Resolving the mentioned equation for a motionless wall relative $W_s(\varphi)$ one obtains

$$[w(\varphi) - w(\varphi_e)]/S_d = (1/2)[(K_w/K_{22})(d/L_w)^2(d\varphi/dx)^2 + (\pi/2 - \varphi_e)^2 - (\pi/2 - \varphi)^2], \qquad (14)$$

where x is the dimensional coordinate along the wedge surface perpendicular to the wall (the distance x is normalized by the wall thickness L_w) and $K_w = K_{11}/6 + (7/24)K_{33}$. So, $d\varphi/dx$ calculated from the measured in the experiment $\varphi(x)$ allows to restore the shape of the potential in the range $\varphi_e < \varphi < \pi/2$.

To find the normalizing factor W for the anchoring potential (depth of the potential well) one has to find the following difference of the quantities found from the experiment

$$W/2 = [w(\pi/2) - w(\varphi_e)]/S_d + w(\varphi_e)/S_d - w(0)/S_d, \qquad (15)$$

where the first term in the right side of Eq. (15) has to be found according Eq. (14) (measurements in the wall) and the second and third terms have to be found from Eqs. (6) (measurements in Cano-Granjean zone between two consecutive walls). The found from the measurements function $w(\varphi)/S_d$ being normalized by the factor W from (15) satisfies to Eqs. (2) what allows to determine l_p from the measurements, for example at the point $\varphi = \varphi_e$ and to check the consistency of the procedure with help of Eq. (13) and the formula

$$\varphi_e - \pi/2 = (\partial w(\varphi)/\partial \varphi)/S_d \tag{16}$$

at $\varphi = \varphi_e$, i.e., at the position of the wall at the wedge. So to fulfill the restoration procedure it is sufficient to perform the measurements at one Cano-Granjean zone.

In the case of a plane layer of varying thickness the consistency of the procedure may be verified by application of the formula

$$l_p = (p/d_j)S_d = -(p/d_j)\partial^2 w(\varphi)/\partial^2 \varphi \tag{17}$$

at the thickness d_j corresponding to the jump.

Note that the described above procedure of the surface anchoring potential restoration for the case of fixed layer thickness (limited by the range $0 < \varphi < \varphi_c$) may be also supplemented by the measurements at the wall in a plane layer based at Eq. (14). The measurements at the layer of fixed thickness supplemented by the measurements at the wall allow to restore the potential in whole range of angles ($0 < \varphi < \pi/2$). However, one has to keep in the mind that obtaining of a motionless wall in the layer demands additional experimental efforts. Another experimental difficulties (related both to a layer of constant and varying thickness) may arise due to a narrowness of the wall [4] which may occur to be less than the coordinate resolution of conventional optical methods. If the resolution of optical methods is insufficient a new direct method of the director orientation determination [7,8] may be applied what gives hope to resolve the coordinate dependence of the director orientation inside the wall.

The options to check consistency of the actual surface anchoring potential restoration for the case of fixed layer thickness are presented by Eqs. (9)–(11).

DIRECTOR DYNAMICS

Other options to restore the surface anchoring potential are connected with experimental studies of the director dynamics at weak surface anchoring. We examine below the dynamics of a pitch jump and motion of nonsingular wall in a planar layer. The corresponding experiments seem to be more complicated than the static ones and include additional parameter of LC, namely the twist viscosity, however, they also allow to restore the actual surface anchoring potential directly from the experiment.

Jump dynamics. If above in the static examinations of the anchoring the temporal characteristics of the jump were irrelevant to the surface anchoring restoration the temporal characteristics of the jump in dynamic studies become very informative on the surface anchoring in general and on the shape of surface anchoring potential in particular. This was demonstrated in papers [4,20] where the calculations of the temporal development of the jump for Rapini–Papoular and B-like model surface anchoring potentials reveal essential difference of the jump dynamics for these potentials.

We shall find below the connection of the temporal variation of the director rotation velocity at the surface with the actual surface anchoring potential following the approach of [4,20] and discuss the potential restoration procedure based at the experimental measurements.

So, we shall examine the jump of the pitch in a CLC layer (with the same surface conditions as above) initiated by variation of the external agent, for example the temperature, which starts at the critical angle φ_c and ends at the angle φ_j corresponding to the equilibrium state for the current value of the temperature. To find an explicit connection of the potential to the jump dynamics we assume like in [4,20] that the jump proceeds in a quasi static regime, i.e., that at any moment in the course of the jump the director distribution in the layer is determined by the static relations between the director distribution in the layer and the current pitch value.

From the Eq. (24) in [20] and Eq. (1) one easily finds

$$\partial \mathrm{w}(\varphi)/\partial\varphi = -\{(\mathrm{d}\varphi/\mathrm{dt})/[(3/\gamma_1 \mathrm{d})\mathrm{W}] + \mathrm{S_d}[\varphi - \varphi_0(\mathrm{T})]\}, \tag{18}$$

where γ_1 is the twist viscosity.

The Eq. (18) is valid for φ changing from φ_c to φ_j. The angular interval (φ_c, φ_j) may include the interval (0, $\pi/2$) or (-0, $-\pi/2$). In this case the Eq. (18) determines $\partial \mathrm{w}(\varphi)/\partial\varphi$ (and $\mathrm{w}(\varphi)$ by a consequent integration) via measured φ(t) (and calculated $\mathrm{d}\varphi/\mathrm{dt}$ from φ(t)) in the whole interval of $\mathrm{w}(\varphi)$ determination. Note that $\mathrm{w}(\varphi)$ is an even function of φ so it is sufficient to determine it in the interval (0, $\pi/2$) or (-0, $-\pi/2$ only). If the interval (0, $\pi/2$) or (-0, $-\pi/2$) is not overlapped by the measured $\varphi(t)$ interval $\partial \mathrm{w}(\varphi)/\partial\varphi$ may be restored in the part of the interval (0, $\pi/2$). Here we assume for simplicity that W (i.e., S_d) is known. Otherwise one has to apply procedure of refining S_d similar to described above. To illustrate at what extent the jump dynamics is sensitive to the shape and the strength of the anchoring potential we present at Figures 5 and 6 the corresponding calculations [20] performed for R–P and B-model surface anchoring potentials.

Motion of a nonsingular wall. Information on the actual surface anchoring potential which may be obtained from the measurements on moving nonsingular walls [2,4,16] is restricted by the angular interval less than (0, $\pi/2$). Moreover, the corresponding experimental measurements look as rather complicated. Nevertheless we discuss here the simplest option for the surface anchoring potential restoration in a limited angular range related to the motion of a flat nonsingular wall studied in [2,4,16]. As it was shown in the cited papers a flat wall is mowing with constant velocity v_s which relates to the difference of the free energies per unit area at the both sides of the wall ΔF through the relation [4,16]

$$v_s = -\Delta F/(\gamma_1 V), \tag{19}$$

where V is, so called, the dissipation integral determined by the formula

$$V = \int (d\O(x,z)/dx)^2]dzdx, \tag{20}$$

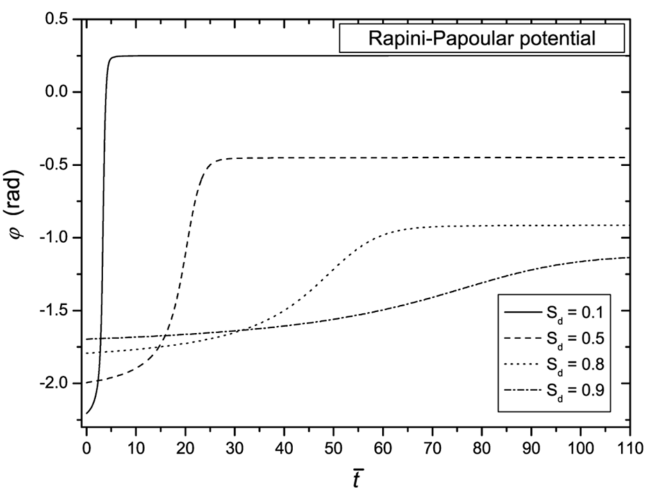

FIGURE 5 Temporal behavior of the director orientation angle φ at the surface during a jump for the R–P-potential at the indicated values of S_d [20].

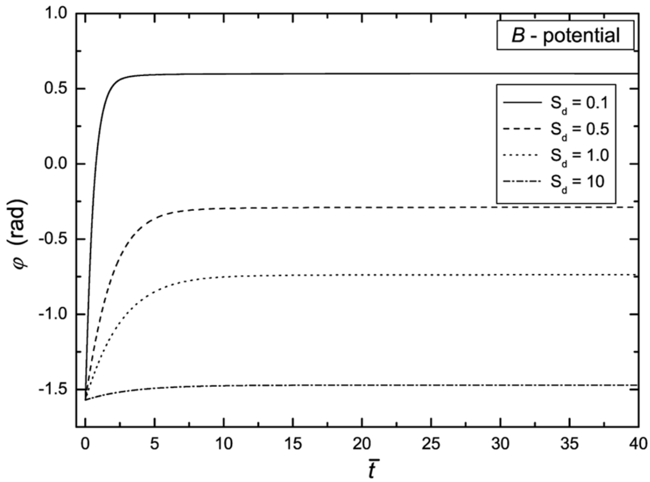

FIGURE 6 Temporal behavior of the director orientation angle φ at the surface during a jump for the B-potential at the indicated values of S_d [20].

where the integration is carried out along whole layer, Ø(x,z) is the local director twist angle (dependent on the coordinate x along the wall motion and coordinate z determining the distance from the layer surface).

The quantity ΔF in (19) depends on the differences of elastic energies per unit area of the layer (second term in the right side of Eq. (1)) and surface anchoring energies at two different values of φ at two sides of a moving wall. The differences of elastic energies may be calculated and inserted into ΔF in (19) what present the opportunity to connect the differences of elastic energies with the velocity v_s. This presents, in principle, an option to restore the actual surface anchoring potential if measurements of the velocity v_s are performed for sufficiently large number of different φ at two sides of a moving wall (different values of $\varphi_0(T)$ in Eq. (1)). However this procedure looks as a too tricky and we point out another more clear option, namely the measurements of v_s versus φ close too φ_e for very low v_s (at almost motionless wall for which $\varphi = \varphi_e$). Remind, that motionless walls occur at the layer thickness equal to $(n + 1/2)p$, i.e., at $\varphi_0(T) = \pi/2$, where n is integer and the equilibrium angle φ_e is dependent on the layer thickness [4].

At the mentioned restrictions Eq. (19) reduces to

$$v_s = -2(\varphi_0(T) - \pi/2)(\partial W_s(\varphi)/\partial\varphi)_{\varphi=\varphi_e}/(\gamma_1 V). \tag{21}$$

Equation (20) shows that for φ_0 close to $\pi/2$ the wall velocity v_s is linearly dependent on $(\varphi_0 - \pi/2)$ and is proportional (with a known proportionality factor) to the angular derivative of the surface anchoring potential at the angle $\varphi = \varphi_e$. Thus, measuring of the proportionality coefficient in the dependence of v_s on $\varphi_0(T)$ (pitch value) for various layer thicknesses and values of pitch one is able to restore through Eq. (20) the derivative of the surface anchoring potential in the interval of φ determined by accessible in the experiment values of φ_e [4]. Note, that applying Eq. (20) for restoration of the surface anchoring potential one has to take into account the dependence of the dissipation integral V (see Eq. (20)) on the layer thickness [4].

EXPERIMENTAL DATA

Let us look briefly what information on the surface anchoring potential can be obtained from the available experimental data. Begin from the paper [19] where the temperature dependence of the director orientation on the surface of a plane CLC layer was measured and the measured data where used to determine the parameter S_d in assumption that the anchoring is described by R–P-potential. The number of experimental points was insufficient to restore the potential without some assumption on the potential shape however it was possible to determine the coefficient in Eq. (7) and to estimate the angle φ at the jump points. One gets from the data presented in [19] the following values: $S_d = 0.14$, $k = 0.21$ and $\varphi_c \geq \pi/10$. The measurements of a nematic layer twisting by mechanical rotation of a cell plate in [13] where the data on layer twisting versus plate rotation, on jump points and hysteresis were presented may be used also to get some information on the surface anchoring potential. The corresponding data are insufficient to restore the potential outside the range of small deviation angles φ, due to insufficiently weak anchoring however it was possible to determine the coefficient in Eq. (7) and to measure the angle φ at the jump points and estimate S_d. One gets from the data presented in [13] the following values: $S_d = 0.12$ for R–P-potential, $S_d = 0.06$ for B-potential, $k = 0.44$ and $\varphi_c \geq \pi/15$. The estimated from the papers [13,19] angular dependence of the potential derivative together with the corresponding dependences for R–P- and B-potentials are presented at Figure 7. Note that the estimated dependences are presented in a limited angular interval due to small values of the angle φ at the jump points. Point out also that the measured values of the angle φ at the jump (and after jump) points (Fig. 10 in [13] and Fig. 4 in [19]) satisfy reasonable well to the self-consistency Eq. (11) for the surface anchoring potential of the shape presented by Eq. (7).

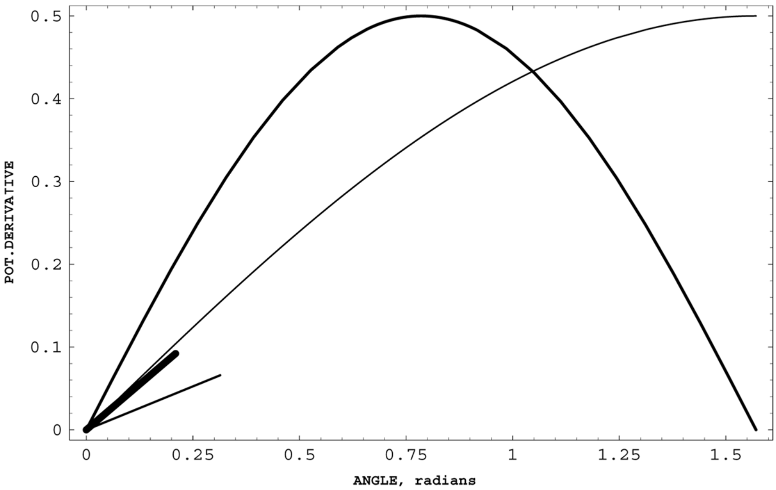

FIGURE 7 Restored from the experiment [19] (thin line) and experiment [13] (bold line) derivative of the normalized surface anchoring potential (see the text after Eq. (2)) versus director deviation angle φ together with the same derivative for R–P (bold line) and B-model potentials.

In the both papers [13,19] the jump angle is essentially less than the critical angle φ_c corresponding to the found parameters in the experiments: for B-potential $\varphi_c = \pi/2$, for R–P-potential $\varphi_c = [\arccos(-S_d)]/2$ [3,13]. The lower estimate of the critical angles corresponding to R–P-potential for the both experiment is $\varphi_c \sim \pi/4$. The most probable reason for this discrepancy between the theory and experiment is the influence of thermal fluctuations [13,21] which reduce the value of the jump angle moving it to φ_e. Note, that the estimated value of the coefficient k in Eq. (7) shows that the bottom of the actual surface anchoring potential well is more flat than for R–P- and B-potentials. The evident conclusion from the presented experimental data is the need to perform new experiments at more weak anchoring or at more thin samples for the restoration of the actual surface anchoring potential.

What is concerned of the experiments on dynamics there are no publications up to now containing data relevant to the discussed procedure of the actual surface anchoring potential restoring.

CONCLUSION

The discussed above progress in the restoring of the actual surface anchoring potential shows that up to date there was no measurements which allow to restore the actual potential in sufficiently large angular range where the difference of its possible functional dependence on the director deviation angle from the easy axis may be distinguished. However due to the progress in obtaining of ultra weak surface anchoring [5,6] and the development of new technique of high resolution of the director field distribution [7,8] the problem of model independent surface anchoring potential restoration in large angular range from the corresponding measurements became quite solvable. Really, the anchoring energy as low as $10^{-7}-10^{-5}\,J/m^2$ was achieved [5,6] and the director orientation space resolution as low as $1-0.1\,$mkm was achieved [7,8]. An additional favorable conditions for the potential restoration in wide angular range may be reached not only by further lowering of the anchoring energy but also by use of more thin sample and more short pitch CLC because the governing parameter of the problem is not simple the anchoring strength but the dimensionless parameter S_d determined in (2).

As the experiment [11,13,19] and the discussed values of the related parameters of the problem show the restoration of the anchoring potential shape up to the critical (jump) angle is achievable by traditional optical measurements in the range of smooth director deviations from the easy axis (in a wedge or at variations of external agent, i.e., temperature, field and so on). What is concerned of the director deviation angles which are connected with a pitch jump (or nonsingular wall) [2–4] the restoration of the anchoring potential in the corresponding angular range looks as a more complicated one from the experimental point of view due to a small wall thickness what demands rather high space resolution of the measurements. As an alternative to the traditional optical methods here may be applied discussed in the previous sections dynamical measurements and already mentioned a new technique of the director distribution measurements [7,8]. Without any doubts the experience of easy axis gliding studies [6,22–26] may be also useful here.

APPENDIX

New Model Surface Anchoring Potentials

For the purpose of easy reference we give below expressions for the Rapini–Papoular (R–P), so called, B-potential recently introduced in [2,20], and the narrow angular width R–P-like and B-like model surface anchoring potentials introduced in [3] (see Fig. 1).

The B-potential [2,20] is given by the formula:

$$W_s(\varphi) = -W[\cos^2(\varphi/2) - 1/2], \qquad \text{if} \quad -\pi/2 < \varphi < \pi/2. \tag{A.1}$$

The period of the B-potential is π, i.e., $W_s(\varphi + \pi) = W_s(\varphi)$.
The narrow B_n-potential ($n > 1$):

$$\begin{aligned} W_s(\varphi) &= -W(\cos^2(n\varphi/2) - 1/2), && \text{if} \quad -\pi/2n < \varphi < /2n, \\ W_s(\varphi) &= 0, && \text{if} \quad -\pi/2n < |\varphi| < \pi/2, \end{aligned} \tag{A.2}$$

and continued periodically to $|\varphi| > \pi/2$ (see Fig. 1), according to the relation $W_s(\varphi) = W_s(\varphi - \pi)$ where $n > 1$ ($n = 1$ corresponds to the B-potential).

The free energy (1) for the B_n-potential accepts the:

$$\begin{aligned} F(T)/W &= -(\cos^2(n\varphi/2) - 1/2) + (S_d/2)[\varphi - \varphi_0(T)]^2 \ \text{if} - \pi/2n < \varphi < \pi/2n, \\ F(T)/W &= (S_d/2)[\varphi - \varphi_0(T)]^2 \qquad \text{if} \, \pi/2n < |\varphi| < \pi - \pi/2n. \end{aligned} \tag{A.3}$$

By a similar way, as B_n-potential, is determined the narrow R–P_n-potential:

$$\begin{aligned} W_s(\varphi) &= -(W/2)\cos^2(n\varphi), && \text{if} \quad -\pi/2n < \varphi < \pi/2n, \\ W_s(\varphi) &= 0, && \text{if} \quad \pi/2n < |\varphi| < \pi/2, \end{aligned} \tag{A.4}$$

and continued periodically to $|\varphi| > \pi/2$ (see Fig. 1b), according to the relation $W_s(\varphi) = W_s(\varphi - \pi)$, where $n > 1$ ($n = 1$ corresponds to the R–P-potential).

The free energy (1) for the R–P_n-potential accepts the following form:

$$\begin{aligned} F(T)/W &= [-\cos^2(n\varphi) + S_d[\varphi - \varphi_0(T)]^2]/2, && \text{if} \quad -\pi/2n < \varphi < \pi/2n, \\ F(T)/W &= (S_d/2)[\varphi - \varphi_0(T)]^2, && \text{if} \ \pi/2n < |\varphi| < \pi - \pi/2n. \end{aligned} \tag{A.5}$$

One has to keep in the mind that the B-potential, being an alternative to the R–P-potential, is some simple and convenient idealization of the physically acceptable surface anchoring potential. Namely, the B-potential has a discontinuous first derivative at the maximum point (edge of the potential well), and thus the curvature is infinitely large. However, one should accept it as a simple model for a class of possible potentials with a very sharp maximum (i.e., very large but finite curvature at the well edge). What is concerned of the R–P-like and B-like surface anchoring potentials with a narrow angular potential well they are natural generalizations of the R–P- and B-potentials which may be useful in the case of a liquid crystal limited, for example, by a single crystal. In this case more than one alignment direction exists, so the widths of anchoring potential well corresponding to each alignment direction have to be less than π. If one alignment direction is much "stronger" than all other ones, it is possible in the first approximation to neglect by all surface anchoring wells except the one related to the "strong alignment direction". As a result one obtains the potential of R–P_n- and B_n-type discussed here.

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